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Cost Analysis of the Departure–En Route Merge Problem

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The technical challenges associated with merging departure aircraft onto their filed routes in a congested airspace environment is discussed. A cost assessment of merge strategies that are based on four-dimensional flight management principles and conflict detection and resolution is given. Several merge conflicts are studied and a cost for each resolution is computed. In addition, the effects of uncertainties on the cost are assessed. In general, altitude hold for a departure aircraft is a less expensive resolution strategy than vectoring.

Introduction

THE Federal Aviation Administration (FAA) has forecasted that the number of commercial air carrier and commuter flights will continue to increase yearly. Unless there are changes made to the air traffic control system, the increase in traffic will result in substantial delays. As a result, the FAA is undertaking a modernization of the air traffic control system. This effort includes a hardware upgrade of the existing air traffic computer system and the development of software tools to complement the air traffic controller's expert skills. The software tools are intended to enhance the air traffic controller's situational awareness and to provide decision support aids for tactical and strategic decisions. The Center terminal radar approach control (TRACON) Automation System (CTAS), which is being developed at NASA Ames Research Center in cooperation with the FAA, is one example of a set of software tools intended to enhance the air traffic controller's ability to schedule and control air traffic through accurate trajectory predictions, scheduling algorithms, and traffic forecasts.[1,2] Concurrently, Eurocontrol, a multinational consortium of western European countries, is developing a modernized air traffic control system to be used throughout western Europe. These tools are intended to assist controllers with the management of air traffic in all phases of flight.

Historically, the emphasis of air traffic control research has been placed on solving problems regarding the handling of arrival traffic, as this has been the primary source of delay and congestion. More recently, the emphasis has moved toward conflict detection and other concepts related to free flight. Free flight is an advanced concept whereby individual aircraft select, and are permitted to fly, optimal trajectories. It is foreseen that the job of the air traffic controller may become more passive, requiring intervention only when conflicts arise. Conflict resolution and detection strategies have been the subject of much research.[3–5] In the United States the development of conflict probe tools has occurred independently at the Center for Advanced Aviation Systems Development (CAASD) and NASA Ames Research Center.

The foundation on which all decision support tools are built is an ability to accurately predict a given aircraft's state in the near future. The necessity of trajectory prediction in a time-based air traffic control system due to the presence of aircraft with and without four-dimensional flight management systems has been discussed.[6] Accurate four-dimensional trajectories enable potential conflicts to be more reliably predicted. As a result the separation between aircraft can be maintained. Four-dimensional guidance and fuel-optimal flight trajectory algorithms have been under development since the mid 1970s.[7] These efforts have focused on using onboard flight management systems to control arrival time through the use of a cost index.[8,9]

The goal of the current research is to study the costs of merging departures with en route aircraft. This research applies the concepts of conflict probing and trial planning to the problem of determining the optimal trajectory that will merge an aircraft from its departure route onto the initial en route segment. The effect of trajectory prediction accuracy on the timing and implementation of the merge is also considered. Software was developed to analyze several different scenarios. For each case, a set of resolution trajectories are generated, and a cost is determined relative to an undelayed, nominal trajectory.

Problem Statement

Prior to departure, all commercial flights file intended routes of flight from their departure airport to their destination airport. For flights that originate at the larger hub airports, part of the filed route may include a route segment from the airport to the first leg of the en route portion of the route. These segments are called standard instrument departure (SID) routes. SID routes are used as the primary routes out of the terminal area airspace. There are two types of SIDs, pilot nav and radar. If a pilot nav SID is filed, the pilot is responsible for navigation along the route. Radar SIDs are a series of vectors given by the controller to the pilot to merge the aircraft into the overhead traffic. It is along the departure route that the aircraft climbs to altitude and, at the controller's discretion, is given one or more advisories on merging into the overhead traffic. However, observation has shown that the departure routes are not necessarily followed.[10] Reasons for this include the filing of new flight plans once aloft, vectoring to provide in-trail separation between departures and/or en route aircraft, or expediting the aircraft's transition to the en route part of the route.

The problem of merging departures into the en route traffic is best described as the resolution of conflicts between departure aircraft and aircraft that are cruising at altitude. The intent of this paper is to analyze strategies that will help the air traffic controller resolve conflicts as they pertain to departures. In our analysis of the departure merge problem, conflicts are referred to as either a crossing conflict or a merge conflict. The resolution chosen by the controller is dependent on the type of conflict being resolved. In both cases a conflict between two aircraft occurs when horizontal and vertical separation are simultaneously less than the legal minimum separations. A crossing conflict is a conflict where aircraft are on paths that intersect only at a single point. A merge conflict occurs when two aircraft trajectories merge onto a common route segment, and a conflict occurs at or near the merge point.

There are some characteristics that are unique to the departure merge problem. The most obvious is the initial speed difference in the aircraft. Because departures are climbing to altitude, their initial speed is much lower than the speed of the aircraft that are already at altitude. The initial altitude separation is also unique to this problem. Speed changes along the climb trajectory and the time needed

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to reach altitude make it difficult for controllers to estimate the aircraft's top of climb; therefore, current conflict resolution strategies tend to be very conservative. As a result, the controller is more likely to utilize altitude separation to ensure that there is not a conflict. Finally, for most merge trajectories, the aircraft tend to be at nearly the same heading. The small difference in headings means that the duration of the conflict may be several minutes long.

The primary function of a merge tool will be to give recommendations to the controller for cost efficient and conflict-free climb trajectories. It is desired to resolve conflicts as early as possible, but, practically, the accuracy of the predicted trajectories determines the optimal time to resolve the conflict. In the field implementation, the merge tool software will compute a four-dimensional trajectory for each aircraft. The trajectories are then compared for conflicts. If a conflict is predicted, the controller is alerted. Action may be taken on initial notification of the conflict, but also may be delayed at the controller's discretion. The controller can choose any of the following resolutions: a temporary altitude restriction (a T altitude), a trial planning vector, a speed change, or a new routing. If a T altitude is chosen, the controller selects the new altitude, and the controller is given the length of time that the departure needs to spend at that altitude to resolve the conflict. Likewise, if a vector is given to the departure, the turn back point is computed. Speed control is also an option, but for reasons given in the following section, we envision this is not a preferred resolution strategy. In any case, the controller should be able to see estimates of the position and time when the aircraft climbs through a given altitude. It is believed that this may be used to help with deciding on the appropriate resolution maneuver.

The motivation for this analysis is the proposed development of a departure merge tool. Our discussions with air traffic controllers indicate that the task associated with merging departures is difficult. Controllers will try to avoid a merge unless it is dictated by the traffic situation. This research assumes that given the proper information and tools, departure aircraft can be successfully merged onto busy routes. The analysis that follows investigates scenarios likely to be encountered by controllers working departure sectors in the center airspace. Because individual sectors can be subjected to periods of congestion due to short term increases in traffic volume, it is appropriate to investigate ways of climbing departures to altitude along conflict-free, minimum cost trajectories. However, this cannot be accomplished unless a study is conducted that looks at the impact of a set of resolution techniques and the cost passed on to the airline. At a minimum, it is also necessary to qualitatively assess the effects of these resolutions on controller workload. In the sections that follow, resolutions as applied to merge and crossing conflicts are evaluated.

Resolution Techniques

Several resolution techniques are available to resolve both crossing and merge conflicts. There are four primary types of resolutions: 1) Applying of speed changes or restrictions during the climb, 2) vectoring the departure aircraft, 3) holding the departure at some intermediate altitude until the conflict has passed, and 4) clearing the departure direct to an alternate waypoint along the route.

Speed Control

One way to resolve a conflict is by increasing or decreasing the speed of one or more aircraft. The use of speed control has been demonstrated in the Descent Advisor as a means to provide separation between aircraft merging at a TRACON cornerpost.[11,12] An algorithm to determine a calibrated airspeed (CAS) given a desired crossing time is presented by Czerlitzki and Kohrs[13] based on an average wind speed over the length of the trajectory, the average climb rate, the current average CAS, the distance to fly, the current time prediction, and the required time at the control point. Also, Adam and Kohrs[14] give a three-step algorithm to determine a descent CAS based on predicted times and required times at two points along the trajectory. However, our analysis shows that speed control is not an efficient means to increase spacing due to limitations on the climb performance of the aircraft. Speed control is used, however, to maintain spacing within a stream of aircraft.

Most climb trajectories consist of an initial constant CAS segment that transitions to a constant Mach segment when the Mach number rises to a value determined by the climb aircraft's aerodynamic characteristics. To adjust the aircraft arrival time we vary the climb CAS while holding the nominal climb Mach constant. As an example, consider a 737-400 with an initial weight of 110,000 lb, an initial altitude of 13,000 ft, and a final altitude of 33,000 ft. The nominal climb profile is comprised of a 280-kn constant CAS segment followed by a constant Mach climb segment at 0.72M. By varying the initial climb CAS from 250 to 310 kn (while holding climb and cruise Mach fixed at 0.72M) it can be shown that for a 200-n mile long trajectory, the aircraft can be expedited by 28 s by increasing the CAS to 310 kn or delayed by 48 s by decreasing the CAS to 250 kn. Alternately, a 757 is a larger aircraft that typically flies faster and higher than the 737. The nominal climb profile for the 757 is to fly a constant CAS segment at 300 kn, followed by a constant Mach segment at 0.80M. With the same initial altitude and speed as before, an initial weight of 210,000 lb, a cruise Mach of 0.82, and a final altitude of 37,000 ft, the 757 can be delayed by 61s or expedited 28 s. Note that the range of times available due to speed control will be a function of the cruise altitude and the length of the route segment. Most climb sectors are probably not of adequate length to allow speed control alone to be used for conflict resolution.

Vectoring

Vectoring the departure is another option to increase separation between aircraft. To compute an appropriate path to resolve a conflict, it is necessary to determine whether the conflicting aircraft will be merging or if the flight paths are only crossing. The reason this is necessary is that the vectoring profile is different depending on which case is being considered. Zhao and Schultz[15] studied optimal conflict resolution trajectories for the deterministic case for cruising aircraft and showed that the optimal resolution to the merge problem was to turn the slower aircraft away from the other and then turn it back. For the crossing conflict, the aircraft are turned toward one another. This prevents a conflict between the same aircraft from occurring farther downstream for some crossing angles. For a crossing conflict, Paielli and Erzberger[3] determine a resolution trajectory by turning one aircraft toward the other (which remains straight and level) until the postresolution conflict probability is less than some acceptable upper bound.

To compute a vector to merge, it is assumed that the departure will merge with the en route traffic at the transition fix. The predicted separation and the required separation are then used to determine a trajectory that will deliver the departure at the fix such that it is a specified distance in-trail to the other aircraft. Based on current practice we assume that the aircraft will make only a 15-deg turn away from the nominal route. Depending on the separation needed, the turn back may require a heading change on the order of 45 deg. The purpose of this procedure is to lengthen the departure's trajectory while maintaining the nominal climb profile. Alternately, if the aircraft are simply crossing paths, the initial turn is computed such that the departure aircraft passes behind the en route aircraft. The departure then returns to its nominal trajectory at the first waypoint downstream from the transition fix.

Altitude Hold

Holding an aircraft at a temporary, or T, altitude is probably the easiest way to resolve either type of conflict. The use of a T altitude is a convenient solution for both the pilot and controller. For the controller, holding at an intermediate altitude requires no predictive insight because the climb can be resumed at an arbitrary time in the future once the conflict has been safely resolved. For the pilot, no vectoring off of the nominal route is required, so workload is minimal. Separation is achieved because at a fixed CAS, the true airspeeds are different at different altitudes (the speed differences are about 7 kn per 1000 ft of altitude separation for the same CAS). Note that the altitude can be considered a form of speed control, although from a pilot and controller perspective, no speed advisories are given or received.

Observation has shown that T altitudes are already used a significant amount of the time by the lower-altitude sector controllers. A majority of these restrictions are likely due to airspace ownership and workload issues rather than conflict problems. The removal of the altitude restriction is highly dependent on the time that the sector handoff is accepted. Therefore, accurate predictions could possibly assist controllers in accepting a handoff sooner because it would be known if a conflict is present in the next sector.

Clear Direct to Next Waypoint

Clearing an aircraft direct to its next waypoint or using a parallel offset route is often used as a means to resolve a merge. There are several cases where this is advantageous. Often two aircraft share a planned route for some short segment before diverging to different destinations. In this case, the most convenient solution, if allowed, is to clear the climbing aircraft to an alternate fix, hence eliminating the conflict. This helps the controller because the controller does not have to monitor when to turn the aircraft back to complete the merge. Instead a single clearance is given and the controller's attention can be directed to other situations that may be more complex. Another reason for clearing an aircraft direct is to separate aircraft with dissimilar speeds. Given a stream of departures coming from an airport, it is likely that they will not be grouped by aircraft type. In fact, there may be a string of slower aircraft ahead of a faster one. In this case, the controller may clear the faster aircraft direct, based on its route and current heading. Tests conducted on the CTAS conflict probe in Denver center show that there were an increased number of aircraft being cleared direct to their next waypoints as a result of the conflict probe's predictions.[16]

Cost Analysis

In this section, we seek to analyze the relative merits of the various conflict resolution strategies through a quantitative cost comparison. In doing this, we examine the change in direct operating cost (DOC) with respect to some nominal trajectory. Throughout the literature, the operating cost is defined to be the sum of the fuel and time costs. This formulation of the DOC is the classical cost function associated with four-dimensional trajectory optimization problems. In reality, additional terms may be present to represent maintenance costs, lease payments, or depreciation,[17] although time and fuel costs are the two largest contributions to the direct operating cost. It is reasonable to assume that the change in direct operating cost of some trajectory with respect of a nominal trajectory is given by

$$\Delta \mathrm{DOC} = c_t \Delta T + c_f \Delta W_f \tag{1}$$

where ΔDOC is the change in the direct operating cost of the aircraft, c_t is the time cost factor measured in dollars per minute, ΔT is the time difference in minutes required to fly the trajectories, c_f is the fuel cost in dollars per pound, and ΔW_f is the difference in fuel burn measured in pounds.

First, we consider what happens when vectoring is used. The purpose of vectoring is to have one aircraft fly an additional distance by turning it away from the nominal route. This directly translates into an additional time to fly ΔT. For an aircraft that is climbing, this additional time would occur at altitude as long as the nominal climb schedule is not altered. If this is the case, then we can assume that the fuel flow $\dot{W}_f$ is constant over the cruise segment. The difference in fuel burn for the two trajectories is $\Delta W_f = \dot{W}_f \Delta T$. Equation 1 then becomes

$$\Delta \mathrm{DOC} = (c_t + c_f \dot{W}_f)\Delta T \tag{2}$$

The increase in cost is then directly proportional to the increase in time that the vector maneuver needs to absorb. For representative numbers consider $c_t = \$8.33$/min, $c_f = \$0.19$/lb. A typical fuel flow for a 737-400 at FL330 i.e., altitude = 33,000 ft is $\dot{W}_f = 85$ lb/min; therefore, a delay of 1 min will yield a cost increase of \$24.74. On the other hand, a 757 at FL370 has an average fuel flow of $\dot{W}_f = 125$ lb/min, yielding a cost of \$32.50 for 1 min of delay.

As a second case, we consider what happens when the departure aircraft is held at an altitude that is lower than the cruise altitude for some period of time. This maneuver is more complex to analyze due to the fact that true air speed (TAS) is a function of altitude and CAS or Mach number. Consider the 737 and 757 examples. In each case, the time and cost to obtain 1-n mile separation will be computed for a range of valid hold altitudes where the conflicting aircraft is flying at cruise altitude at a specified Mach number.

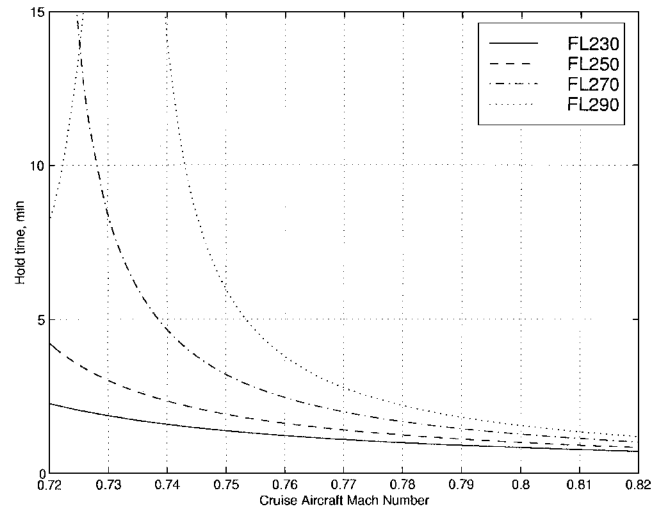

Fig. 1 Hold time to achieve 1-n mile separation (737).

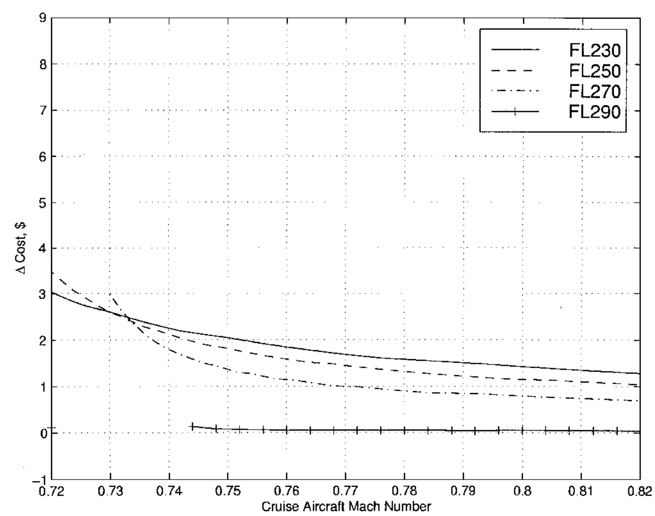

Fig. 2 Δ cost to achieve 1-n mile separation (737).

For the first case the 737-400 is to fly an optimal climb schedule [280 kn CAS (KCAS)/0.72M], and then cruise at FL330 at 0.72M. Figure 1 shows hold time as a function of hold altitude and the cruise aircraft's Mach number. The minimum hold times for the 737 are at FL230. Note that at FL290, there is no solution if the cruise aircraft is slower than 0.74M. The peak on this curve occurs because at 280 KCAS and FL290, the TAS is the nearly same for an aircraft at FL330 and 0.74M. In this case the hold time approaches infinity because the two aircraft are at the same ground speed. Using previously cited values for c_t and c_f, the costs associated with these hold times are shown in Fig. 2. The minimum cost is to hold at FL290 when the en route aircraft is 0.745M or faster. This shows almost no cost increase over the nominal trajectory. This is because, for our aerodynamic model, the economy specific range (defined as the specific range optimized for the cost parameters chosen) at these two points in the aircraft's envelope are nearly the same. If the en route aircraft is slower than 0.745M, then one of the lower altitudes may be selected, although it will be at a higher cost than if the aircraft were holding at FL290.

The comparable times to hold for the 757 are shown in Fig. 3. For aircraft that are cruising at 0.782M or less, the minimum time

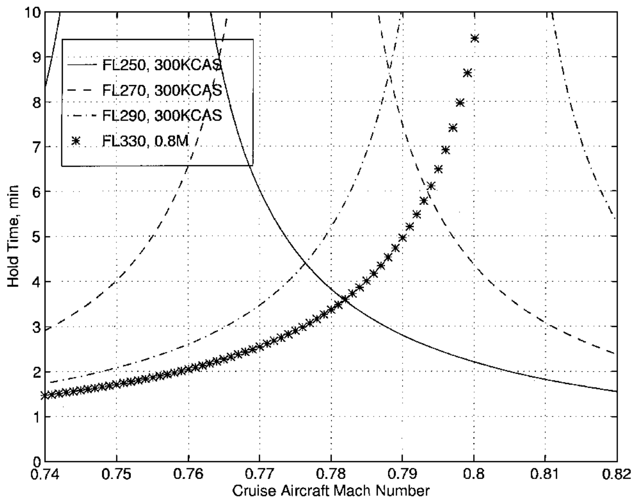

Fig. 3 Hold time to achieve 1-n mile separation (757).

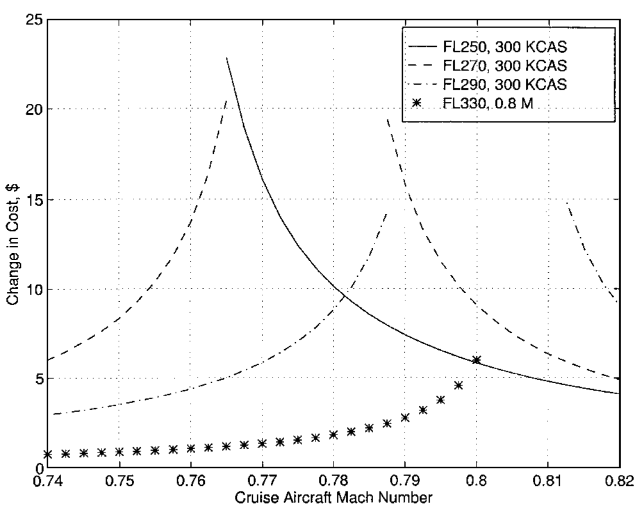

Fig. 4 Δ cost to achieve 1-n mile separation (757).

to hold is at FL330. This corresponds to the 757 passing in front of the cruising aircraft. However, if the en route aircraft is faster than 0.782M, the minimum hold times occur at FL250. At this altitude, the 757 is slower than the en route aircraft; therefore, the en route aircraft passes in front. These altitudes are what would be expected in terms of minimizing the time to hold. At FL330, when the 757 is passing the en route aircraft, this represents the largest speed difference. The same is true at FL250 when the cruise aircraft is passing the 757; the speed difference is maximized, hence, the hold time is minimized. The costs that correspond to the hold times are shown in Fig. 4. Figure 4 shows that the minimum cost hold altitude is FL330 if the cruise aircraft is at 0.8M or slower. This is despite the hold times conceivably being longer than for lower hold altitudes. The higher speed and the improved fuel burn of the aircraft at FL330 results in a higher cost savings. If the en route aircraft is cruising faster than 0.8M, then the minimum cost strategy is to hold at FL250 and to let the en route aircraft pull ahead.

For this calculation, we are assuming a zero wind. If there is a difference in wind speeds between the hold and desired cruise altitudes then the effect of the wind will be to change the relative ground speed of the two aircraft and, hence, to change the hold time and possibly the optimal strategy to minimize cost. Also, it should be realized that the conflict prediction horizon will ultimately decide the hold altitude. If accurate time predictions are not possible, the departure aircraft may have already passed through the optimal hold altitude before the conflict is detected. Then it will have to hold at another altitude that is not optimal.

What we have shown is that in terms of cost, altitude hold is generally going to be better than vectoring. This is especially true when there are large differences in true airspeed between the departure aircraft and the en route aircraft. The advantages to doing an altitude hold are that it is simple and cost effective, and the controller can use the speed differences of the aircraft to obtain a favorable in-trail sequence, that is, the controller can put a slower aircraft behind a faster aircraft. On the other hand, vectoring may be preferable if the true airspeeds of the two aircraft are close, or if the controller wants the two aircraft in-trail at some specified point.

Results

This section presents the results of a cost study of the resolutions discussed for a particular encounter geometry and location. Several scenarios are analyzed that are believed to be typical situations that controllers in every center see on a daily basis. The metric used to quantify each trajectory is the change in direct operating cost [Eq. (1)] with respect to a nominal cost for an undelayed trajectory to a fixed point. For the cases studied here, the value of c_f is \$1.25/gal or \$0.19/lb and the value of c_t is \$8.33/min or \$0.14/s. These values were selected for the cost coefficients because it is believed that they represent typical values.

Figure 5 shows the airspace under consideration for this study. This particular set of routes is based upon the Denver PLAINS SID. We believe that this airspace is representative of the departure merge problem in general. Several jetways intersect at the Goodland (GLD) intersection including a major east-west jetway (J80), and so merging and crossing aircraft in this area are common. For our study, only aircraft constrained to their routes were considered.

Merge Conflicts

This section focuses on the accurate delivery of a departure aircraft such that it is merged to fly in-trail with one or more en route aircraft. It is assumed that the en route aircraft are established eastbound on a jetway and will remain so until well past the merge point. The departure aircraft is assumed to follow the same jetway eastbound after joining the jetway at the GLD VORTAC. It is assumed that the trajectory predictions are exact. A quantitative discussion of the effects of trajectory prediction errors on the advisory decision follows in the next section.

We present several cases where a departure aircraft, assumed to be either a 737-400 or a 757-200, has departed Denver International Airport (DEN). The departure aircraft is initially at an altitude of 12,900 ft with a true airspeed of 325 kn, and a heading of 108 deg measured positive clockwise from north. The 737 has a filed cruise altitude of 33,000 ft and a cruise Mach of 0.72 (418 KTAS), which represents the cruise Mach optimized for the cost parameters that we have chosen. The initial aircraft weight is assumed to be 110,000 lb and will fly the following climb profile: a constant 280 KCAS until the aircraft reaches 0.72 Mach. The 757 has a filed cruise altitude of 37,000 ft and a cruise Mach number equal to 0.8 (459 KTAS). This is the optimum Mach for our cost parameters and cruise altitude. The initial weight of the 757 is 200,000 lb. The initial positions and true airspeeds of the en route aircraft are selected to create a conflict between the departure and the en route traffic. The goal is to find the most cost efficient merge trajectories. Accurate performance models of the 737-400 and the 757-200 are used to determine the time and fuel burn. The performance models used in this analysis were taken directly from the CTAS aircraft performance data. Winds were not modeled.

737 Example, En Route Aircraft Flying In-Trail: Case 1. The first case is characterized by two aircraft flying in-trail at cruise altitude with a separation of 10 n mile and a common true airspeed of 468 kn. It is predicted that the departure aircraft (A in Fig. 5) will arrive at GLD in 17.7 min and that the en route aircraft (B and C) will arrive at GLD in 16.6 and 17.9 min, respectively. Minimum separation occurs in 11.5 and 19 min, respectively. Two different resolution maneuvers are examined. These are altitude hold and path stretch. Speed control is not used because it does not provide enough variation in the arrival time to allow the departure to achieve separation. It was assumed that the en route aircraft, because they were flying in-trail with identical speeds, could not be separated. Costs for the two resolutions as compared to an undelayed trajectory are shown in Table 1. Holding the departure aircraft for 28 min at FL290 gave the necessary separation to place aircraft A behind aircraft C. This is the least expensive of the two resolutions for this case. For the altitude hold the slight cost increase can be attributed to the fact that only 29 lb of extra fuel is used. This is offset by the fact that the time cost is negative due to the higher true airspeed when the aircraft is at FL290. Vectoring the aircraft is the more expensive maneuver due to contributions from the extra time needed to merge the aircraft and the extra fuel burned during this time. This trajectory delays the arrival of the departure at the merge point by approximately 1 min. To fly this extra distance, it takes approximately 83 lb of fuel, and so the fuel cost as compared to the altitude hold is more than doubled. Unlike the altitude hold, both the increase in the time and the fuel used directly increase the cost.

737 Example, Single En Route Aircraft: Case 2. The second case considers a situation where there is a single en route aircraft that is in conflict with the departure. The departure (aircraft A) arrives at GLD before the en route aircraft (aircraft B), but is almost 30 kn slower. The arrival times at GLD are separated by about 10 s. The cost analysis is shown in Table 1. The situation is resolved by putting A behind B to prevent B from being held up. The amount of delay necessary explains the increase in cost over the examples discussed. The most practical solution is to take aircraft A off of its nominal route and place it on a route parallel to its filed route to let B pass. Selection of this route will directly influence the cost of the trajectory. The cost of offsetting the route is a $12.80 decrease over the nominal, undelayed trajectory. Note that the cost of this trajectory may increase at some point downstream if the aircraft is returned to its nominal route. In fact, this parallel offset route may be used in any of the merge cases because we are discussing a fixed set of initial conditions for the 737 aircraft in all cases, and it represents a simple and viable solution in the eyes of a controller. Practically, this option is frequently used to allow one of the conflicting aircraft to proceed directly to a waypoint that is further along the route.

Table 1 737 cost comparison

Case	Altitude hold	Vector	Reroute
1	$5.49	24.04	——
2	$4.35	22.29	−12.80
3	$0.13	6.34	——

Case 3. The third case involves a situation where there is a conflict present, but the resolution will only require a fraction of the separation needed for the previous examples. In this case the departure arrives at GLD in 17.73 min, and the en route aircraft in 17.03 min. Therefore, we expect to see a lower cost penalty associated with the vectoring maneuver as compared to the previous cases. If the departure aircraft is held at FL290 for 13 min, this results in a $0.13 cost increase due to a 12-lb increase in fuel and a 16-s decrease in the time to fly. On the other hand, the vector maneuver (Table 1) is only a $6.34 increase. This is much lower than the previous situations where the increases were on the order of $22. This shows that for the 737 to merge with faster traffic, it is more beneficial to employ an altitude hold restriction.

757 Example: Case 1. The first case involves a 757 departing DEN and an aircraft cruising at FL370 and 463 KTAS. Because the 757 is to cruise at a slower speed, it should be positioned in-trail of the cruising aircraft. The en route aircraft crosses GLD in 16.2 min and the 757 in 16.5 min. Altitude hold was applied by clearing the aircraft to FL250. This altitude was chosen because the speed difference between the en route aircraft and the 757 was 32 KTAS. If FL330 had been chosen, the 757 and the en route aircraft would have been at nearly the same true airspeed. As a result, the aircraft would not have separated, thereby permitting the 757 to resume its climb to FL370. The aircraft remained at FL250 for 6 min and the cost of the trajectory increased by $19.53. The incremental fuel burn was 82 lb and the delay was about 27 s. On the other hand, the vectoring maneuver resulted in a cost of $13.80. The 757 used 52 lb of fuel and was delayed by 26 s. The cost increments are shown in Table 2.

Case 2. For the second case the en route aircraft is at a speed of 448 KTAS, which is 10 KTAS slower than the 757 at cruise altitude. The en route aircraft is predicted to arrive at GLD in 17.1 min. Application of the altitude hold maneuver at FL330 for 12 min

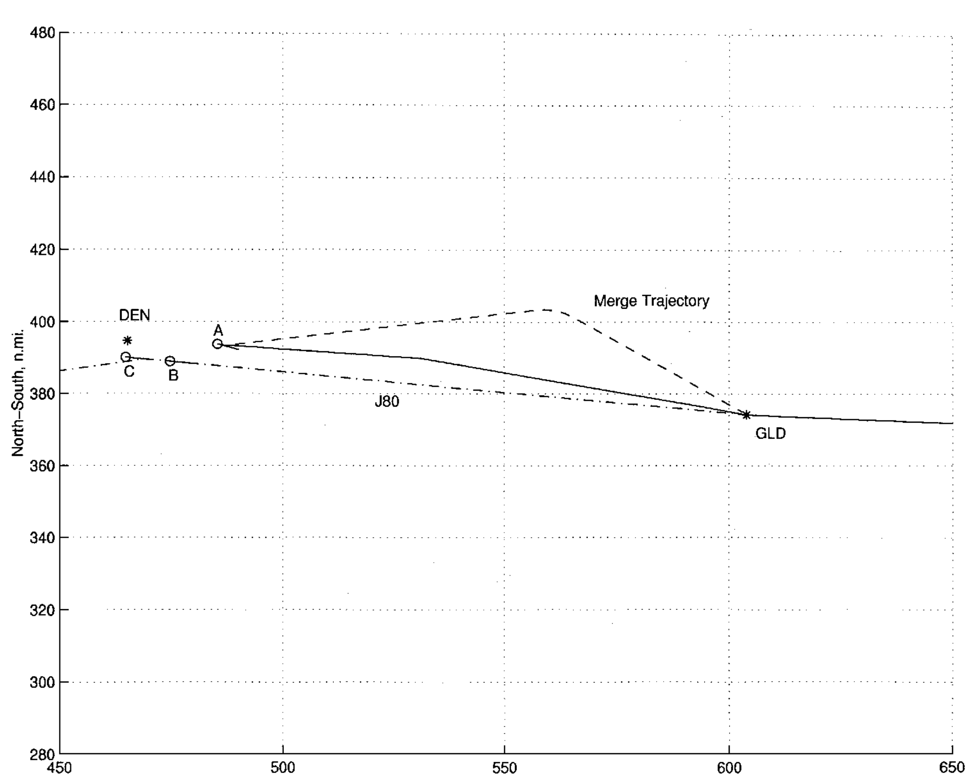

Fig. 5 Path stretch resolution for 737 case 1.

Table 2 757 cost comparison

Case	Altitude hold	Vector	Reroute
1	\$19.53	13.80	——
2	\$8.76	35.22[a]	−12.36

[a]With a 10-kn speed reduction at cruise altitude.

resulted in a cost increase of \$8.76. The aircraft burned 53 lb of fuel, but arrived at the trajectory end point 10 s before the nominal trajectory. A vector maneuver will be unacceptable because of the speed advantage over the en route aircraft at altitude. This speed difference would result in the 757 overtaking the en route aircraft shortly after reaching its cruise altitude. If the cruise speed of the 757 is adjusted to that of the en route aircraft, the cost increase will be \$35.22. As an alternate option, the controller may decide to reroute the departure to avoid a speed adjustment. The cost increment to reroute is a \$12.36 improvement over the nominal trajectory. Table 2 shows the cost comparison.

Effects of Uncertainties on Cost

The analysis presented in the preceding section showed the costs of various resolution maneuvers for a conflict that was about 15 min into the future. These costs establish a baseline of what would be possible under an ideal situation where aircraft trajectories could be computed exactly. The effect of uncertainty on the trajectory is to transform the conflict resolution problem from a deterministic problem to a stochastic one. Therefore, the uncertainty in an aircraft trajectory needs to be considered when determining conflicts. If the uncertainties in the predicted trajectories are known, an estimate of the conflict probability between a pair of aircraft can be determined.[3] The probability estimation algorithm developed by Paielli and Erzberger[3] assumes that prediction errors are normally distributed with zero mean. The along-track rms error and the cross-track rms errors are parameters for the estimation algorithm and determine the covariance. To compute conflict probabilities for our analysis, the algorithm developed by Paielli and Erzberger[3] is followed.

A merge trajectory is usually characterized by small encounter angles between the aircraft. One important conclusion reached by Paielli and Erzberger[3] is that there appears to be a cross correlation of the position errors for two aircraft in cruise on nearly identical headings. The cross correlation of errors is a result of the two aircraft being affected by a common (and uncertain) wind component. The effect of the cross correlation is to reduce the error in the position difference as compared to an individual aircraft's position error. It is not known whether a merge conflict between a climbing aircraft and an aircraft in cruise would have such a cross correlation due to differences in winds aloft at the two altitudes and the speed of the aircraft. There is probably little cross correlation when the aircraft are separated by large altitudes (greater than 10,000 ft); however, as the departure aircraft approaches its cruise altitude, the errors probably become more correlated. We will assume that the position errors are uncorrelated throughout the encounter. Figure 6 shows the conflict probability between a departure and an aircraft at cruise altitude. In Fig. 6 it is assumed that the along-track error grows at rate of 45 kn over the entire departure trajectory. This value of the along-track rms error represents the accuracy of the current departure trajectory prediction capability.[10] The along-track error growth rate for the aircraft in cruise is 15 kn. Each aircraft has a constant cross-track error of 1 n mile. The ground tracks intersect at an angle of 15 deg. The departure's initial conditions are 13,000 ft and 280 KCAS. The aircraft climbs according to a 280 KCAS/0.74 Mach climb schedule, and has a cruise Mach of 0.74 at FL330. The cruise aircraft is at FL330 at 0.78M. Each curve corresponds to a minimum predicted separation. It is assumed that the conflict occurs at a fixed point that occurs a short distance after the departure aircraft reaches cruise altitude. The vertical uncertainty is not considered because the conflict occurs at the cruise altitude. Figure 6 shows that the conflict probability increases rapidly as the aircraft approach minimum separation. For a predicted minimum separation of 0 n mile, the conflict probability reaches 0.8 approximately 5 min before the conflict. Operationally, if the conflict probability is larger than 0.8, it is likely that the conflict will occur and controller intervention is necessary. (Data available online at http://www.atm-seminar-97.eurocontrd.fr/erzberge.htm.)

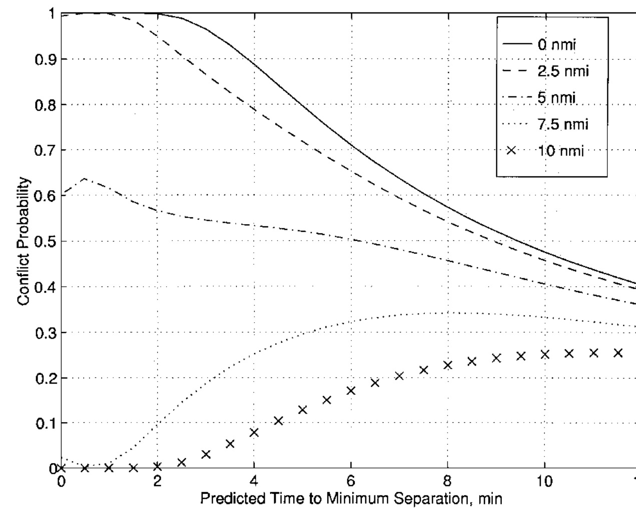

Fig. 6 Conflict probability between a departure and en route aircraft.

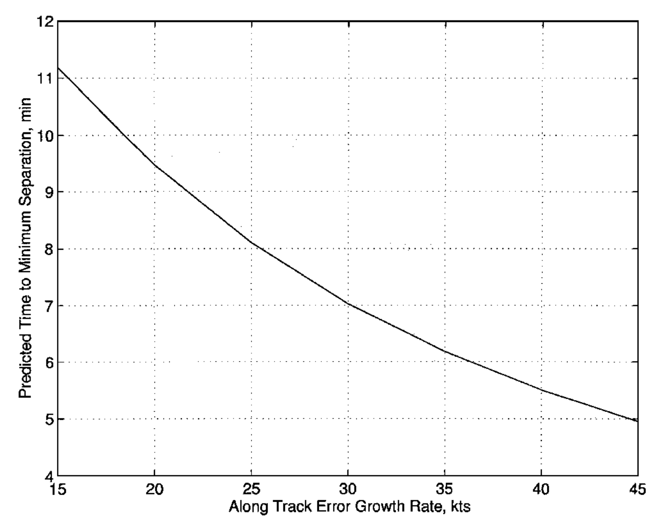

Fig. 7 Conflict alert time (P_c = 0.80) as function of error rate.

Figure 7 shows the effects of improving the departure trajectory prediction on decision time. The decision time is defined as the time remaining to minimum separation when a controller will initiate a resolution maneuver. Figure 7 shows, as a function of along-track growth rate, the time that the conflict probability reaches a value of 0.8 for a predicted separation of 0 n mile. This time will also be used as the decision time. For the current capability of 45-kn rms along-track error, the decision time is 5 min. This improves to approximately 11 min for an along track error of 15 kn.

For a given merge conflict, the resolution cost is invariant with respect to decision time. This can be seen as follows. Consider a vector maneuver to resolve a projected conflict at cruise altitude. With a fixed climb profile, the departure aircraft will require the same amount of delay independent of when the maneuver is initiated. What will differ with decision time is only the size and location of the initial and final turns. The cost however will stay the same. Similarly, for an altitude hold resolution, the cost is invariant until we pass though the desired hold altitude. If the decision time is short and the climbing aircraft has passed the "optimal" hold altitude before the resolution is initiated, then the cost and hold time will be increased.

Conclusions

A quantitative analysis for the problem of merging departures into the en route airspace has been addressed. Maneuver costs when the departure aircraft is approximately 15 min from the merge are evaluated as this represents the ideal capability of a future air traffic control automation system. Of the two most common means of achieving separation, altitude hold appears to be more cost effective than vectoring. Initiating altitude hold is simple, but long hold times may be required to achieve sufficient separation before the aircraft is able to resume its climb to altitude. Vectoring is more complicated because a turn-back point has to be determined. Although vectoring costs more, it may be a desirable solution if the merge is required to occur due to traffic constraints. Speed control alone is generally not a viable method of increasing separation because the initial speed difference between the en route aircraft and the departure aircraft is so large, and the length of the departure route is insufficient to allow a small speed difference to have a major effect on separation. However, speed control is viable if only small changes are required or for maintaining separation within an en route stream or a departure stream.

Rerouting the aircraft was also briefly addressed. This option avoids the merge altogether by establishing the departure on a new route. Rerouting can be used in lieu of vectoring and altitude hold anytime, but is particularly useful when speed differences may result in a loss of in-trail separation after the merge (assuming no en route speed adjustments). Cost savings may be significant depending on the route chosen.

The effects of improving the trajectory prediction are also discussed. It is shown that with current capabilities a conflict between a departure and an en route aircraft cannot be accurately predicted until 5 min prior to the time of minimum separation. We show that for merge conflicts, the cost change is invariant with respect to decision time. Improving the prediction capability will allow the controller to know whether a conflict is likely so that a resolution may be attempted earlier. Also, by improving the prediction capability the controller may be able to rule out anticipated future conflicts and can eliminate unnecessary altitude holds or vectors.

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